

## Wittgenstein and the philosophical significance of not solving the paradoxes

The standard question about the semantic paradoxes is *how* we should solve them. Wittgenstein raised a different question: *whether* we should solve them. In this paper, I argue that we have two reasons to take the question raised by Wittgenstein seriously. First, reflecting on the question posed by Wittgenstein might free us from a philosophical ideal, the assumption that we should reason according to strictly valid logical principles, in the sense of Hofweber (2008, 2009). Second, reflecting on Wittgenstein's question might lead us to realize a possibly obvious, but important, point: the fact that several logical principles are jointly inconsistent does not show that one of them is more problematic than the others, for the same reason why the fact that several plans of action are jointly inconsistent does not show that one of them is more problematic than the others.

### 1. Introduction

How to solve semantic paradoxes like the Liar paradox or Curry's Paradox is an open problem. Different logicians defend different solutions to the semantic paradoxes (see Murzi and Carrara 2014 for an overview). However, those involved in the project of solving the paradoxes apparently share an assumption: that it is important to solve the paradoxes. That assumption is rarely challenged, but it *might* be challenged. Ludwig Wittgenstein might be read as challenging it:

If anyone should think he has solved the problem of life and feel like telling himself that everything is quite easy now, he can see that he is wrong just by recalling that there was a time when this "solution" had not been discovered; but it must have been possible to live *then* too and the solution which has now been discovered seems fortuitous in relation to how things were then. And it is the same in the study of logic. If there were a "solution" to the problems of logic (philosophy) we should only need to caution ourselves that there was a time when they had not been solved (and even at that time people must have known how to live and think).

Wittgenstein 1980, p. 4e

I read this passage as a suggestion to downplay the importance of solving the paradoxes. A related point that Wittgenstein made is that paradoxes are *harmless*; they do not pose any special problem for our standard linguistic practices:

Is there any harm in the contradiction that arises when someone says: "I am lying. - So I am not lying. - So I am lying. - etc."? I mean: does it make our language less usable if in this case, according to the ordinary rules, a proposition yields its contradictory, and vice versa?

Wittgenstein 1978, Appendix III, 12, p.120

The two ideas fit together nicely: if paradoxes do not threaten our linguistic practices, we do not need to solve them.

In this paper, I argue that we have two reasons to take seriously the question whether we really need to solve the paradoxes.

First, reflecting on such a question might free us from a philosophical ideal, the assumption that we should reason according to strictly valid logical principles, in the sense of Hofweber (2008, 2009).

Second, reflecting on Wittgenstein's question might lead us to realize a possibly obvious, but significant, point: the fact that several logical principles are jointly inconsistent does not show that one of them is more problematic than the others, for the same reason why the fact that several plans of action cannot be jointly executed does not show that one of them is more problematic than the others. The following two sections of the paper elaborate upon these two themes.

The goal of this paper is to argue that a certain reading of Wittgenstein suggests a legitimate and interesting point, not to carefully reconstruct Wittgenstein's position concerning the paradoxes. However, in the last section of the paper, I argue that the points that I make in this paper are in harmony with many remarks made by Wittgenstein concerning the right attitude to take with respect to the paradoxes.

## **1. The ideal of deductive logic**

Semantic paradoxes have the form of a piece of reasoning such that each of its steps seems perfectly acceptable, but whose conclusion is unacceptable. It is usually assumed that a solution to the semantic paradoxes should do at least one thing: isolate the wrong inferential step or the wrong premise in the reasoning that leads from apparently acceptable premises to an unacceptable conclusion through apparently correct inferential steps (see Haack (1978, 138-139)).

It is a good question how exactly we should reconstruct the paradoxes. What are the inferential steps? What are the assumptions involved? The problem with these questions, as Visser (1989, 624) notes, is that "What the real assumptions involved in the proof are, will depend on what the true solution of the paradox is". A reconstruction of Curry's paradox like the one offered in Hofweber (2009), for instance, gives the impression that the only principles involved in the paradoxical derivation are the rules of classical logic and the 'naïve' rules for the truth predicate. However, according to other reconstructions of the Curry/Liar paradox (see Ripley 2013), we should recognize that classical logic is perfectly compatible with the naïve truth rules and that the paradox is triggered

by an additional assumption: the transitivity of the relation of logical consequence (which Ripley does not count as part of classical logic).

However interesting the debate about how to reconstruct the semantic paradoxes could be, I do not want to enter into it. The point I want to draw attention to holds even if we admit that the task of reconstructing the paradoxes is connected to the task of solving them. For the purpose of this paper, I am happy to add to the list of the principles that are involved in the paradox the usual suspects: rules of inferences for the connectives, naïve rules for the truth predicate, structural properties of the relation of logical consequence, etc. I am also not interested in discussing the question whether the rules governing the truth predicate should count as logical principles (Priest 2007, Cook 2012): I will speak of semantic/logical principles to remain neutral.

The point I *am* interested in is: no matter how you reconstruct the paradoxical reasoning, none of the assumptions involved stands out as clearly incorrect; on the contrary, each one of them seems perfectly natural and correct. If solving the paradoxes means finding the wrong step in the paradoxical reasoning, then we did not solve the paradoxes (yet). However, our failure to solve the paradoxes did not jeopardize our linguistic and inferential practices. It did not make “our language less usable” as Wittgenstein said in the quote above. How is that possible? The point I want to elaborate upon might be presented in the form of (an admittedly crude) argument:

- P1. If something constitutes a real problem, failure to solve it would have undesirable consequences.
- P2. Failure to solve the logical paradoxes did not have any undesirable consequence.
- C. The logical paradoxes constitute no real problem.

Coming to the same point from a different angle. The reaction of ordinary reasoners in front of the paradoxes is what Hofweber (2008) rightly calls the *natural* reaction: rejecting the conclusion of the argument without rejecting *any* of the rules involved in the paradoxical reasoning, that is doing exactly the opposite of what a solution to the paradoxes should do, according to Haack (1978, 138-9). The natural reaction worked: how is that possible?

The solution to the problem of accounting for the success of the natural reaction, in my view, lies in a distinction that Hofweber (2008) makes. We can distinguish two senses in which a rule of inference can be said to be *valid* (see also Hofweber 2009, p. 18):

1. *Strict* validity: each instance of the rule is truth preserving
2. *Generic* validity: instances of the rule are in general truth preserving, but there might be exceptions

Generic statements tolerate exceptions, which means that generic validity is weaker than strict validity. Despite this, its force should not be underestimated. According to (2), the sense, in which we can say that rules of inference are valid, is the same sense in which we ordinarily say that bears are dangerous. When we say that bears are dangerous, we mean it in a generic sense: we don't completely rule out the possibility of a non-dangerous bear. But this does not mean that properly speaking bears are not really dangerous: they are, and it is a good policy to run away from them. Similarly, saying that rules are valid in a generic sense doesn't mean that they are not really valid: they are, and we are entitled to infer according to them.

The first point I want to stress is that the recognition of the generic validity of our rules of inference offers a solution to the puzzle of the efficacy of the natural reaction. According to the generic conception of validity, the problematic instances of the rules are exceptional cases encountered only in the philosophy room, which do not threaten the validity of the rules of inference in normal cases. Normal cases are *most* cases, which means that the rules are valid in the generic sense and we are entitled to believe the conclusion of an argument that relies on them. This is why the natural reaction works: setting aside few exceptions, ordinary rules of inference are valid and that is enough to make them *usable*.

The way in which generically valid rules can be used is as default rules. In default reasoning we are entitled to believe the conclusion of a valid inference, but we are also allowed to take back the conclusion if further information emerges and disproves it. Good default rules need not be strictly valid.

They still need to be generically valid, though. Just consider how many troubles would cause an inferential practice based on affirming the consequent (AC). Surely, there are instances of AC that are truth preserving, but they are hardly the norm: AC is not valid and thus we should not reason according to it. If our rules of inference were not generically valid we would not be entitled to believe the conclusions we draw applying them.

If our rules were not generically valid, revising them would be mandatory. This suggests that in order to account for the fact that *not* revising logic did *not* cause troubles we should recognize that our ordinary rules of inference are generically valid. The generic validity of our logical rules should thus be common ground.

Logical paradoxes showed us that our rules could not be all strictly valid. The natural reaction to the semantic paradoxes consists in simply accepting the situation and learning to live with valid rules that are not strictly valid. Logical revisionism, on the other hand, is the attempt to replace our only-generically-valid rules with strictly valid rules. This difference should not obliterate a point of contact between the two views: just as friends of the natural reaction admit that our rules are *not* strictly valid, so logical revisionists should concede that they are nonetheless generically valid.

In fact, they do concede that. Graham Priest (2006, Ch. 8), for instance, while advocating the adoption of a non-classical (paraconsistent) logic, addresses the problem of accounting for the success of classical logic in ordinary circumstances, despite its invalidity. His answer is that classical rules are quasi-valid, i.e. valid in all cases in which we are dealing with propositions that are not both true and false (so-called *dialetheias*). In consistent contexts, in which no *dialetheias* are involved, Priests' paraconsistent logic behaves like classical logic. Priest recognizes that consistent contexts are the norm: *dialetheias* are *very* few. So it seems that even Priest should accept the generic validity of classical rules: these rules are valid in general, even though there can be few instances of such rules (those involving *dialetheias*) that are not truth-preserving.

Priest's use of expressions such as "presuppositions" (2006, p. 115) and "default assumptions" (p. 117) when talking about classical logic is telling: as we have seen, generically valid rules can be successfully employed in default reasoning. And Priest seems happy to concede that ordinary reasoning can be safely employed in default reasoning.

Once we recognize that our logical principles are *all* generically valid and that generic validity is all we need for our linguistic and logical practices to be in order, we might free ourselves from what Hofweber (2008) labels *the ideal of deductive logic*, the assumption that our rules of inference need to be strictly valid to be valid at all.

In this section I argued that (i) reflecting on the question raised by Wittgenstein's remark, whether we should really solve the paradoxes, suggests a puzzle, the puzzle of explaining the success

of the natural reaction to the paradoxes and that (ii) the best solution to this puzzle is to abandon the ideal of deductive logic.

Hofweber (2009, p. 9) himself uses an argument virtually identical to the one I reconstructed in this section to motivate the abandon of the ideal of deductive logic. I think the argument deserves consideration and that we should credit Wittgenstein for suggesting it.

Hofweber presents additional arguments to support the abandonment of the ideal of deductive logic (Hofweber 2007, 2008), including one based on revenge paradoxes, one based on the impossibility of proving the strict validity of one's favorite rules of inference (see Field 2008) and one based on the existence of apparent counterexamples to standard rules of inference such as modus ponens (McGee 1995). I will not discuss these arguments in this paper. I am content to point out that we should credit Wittgenstein for the suggestion of the argument against the ideal of deductive logic based on the efficacy of the natural reaction.

In the next section I want to focus on a different way to defend the rationality of the natural reaction.

### **3. Semantic paradoxes and collective tragedies**

In this section I am going to use the notion of *collective tragedy*, introduced by Agustin Rayo, to offer an analysis of the semantic paradoxes. Collective tragedies, in Rayo's definition (Rayo 2019, p. 67), are situations where several agents all act rationally and yet the result of their collective action is bad for everyone. Something goes wrong, but there is no one to blame. I want to suggest an analogy between this kind of situations and the case, exemplified by the paradoxes, in which several logical principles cannot be coherently combined together. Also in this case, I suggest, the failure to combine the principles does not indicate that one of them is more problematic than the others.

I will use a definition of collective tragedy slightly different from that of Rayo. A *collective tragedy*, for me, is a setup where several individually consistent plans of action cannot be combined in a consistent plan (a plan of action is consistent when the assumption that such a plan has been executed is consistent). Yablo (2000) presents a very simple example of a collective tragedy in my sense: if my plan is to choose a positive integer larger than yours and your plan is to choose a positive integers larger than mine, then our individually coherent plans cannot be coherently combined.

Here is a more elaborated example, also due to Yabo, of collective tragedy in my sense. Just like Rayo's examples, it involves a backward omega-sequence: a sequence composed of denumerably many agents where agent number  $n+1$  precedes agent number  $n$ , so that agent zero is the last element of the sequence, agent one the penultimate, etc. Suppose that the plan for each agent is: to choose a (natural) number larger than the number chosen by her predecessor. Each individual plan of action can be executed: if agent  $n+1$  chooses  $m$ , then agent  $n$  can choose  $m+1$ . However, the supposition that all the individual plans have been executed is inconsistent, given that it entails the existence of an infinitely descending chain of natural numbers, starting from the number chosen by agent 0, continuing with the number chosen by agent 1, the number chosen by agent 2, ...

It is worth noting a feature of this example: suppose that the initial collective strategy of the agents is to carry out each of their individual plans. Realizing that such a strategy is inconsistent, they might look for a different strategy, that of executing only some individual plans. The natural thought is to try to maximize the number of individual plans that are executed. The bad news is that this is impossible: the individual plans of a sub-group of agents can all be realized exactly when there are finitely many agents in that sub-group; but there is no larger finite set of agents. However many plans are executed, it could have been possible to execute more. If a solution to the problem of consistently combining the original plans consists in choosing only a finite number of plans to execute, then there is no optimal solution to the collective tragedy under consideration. Some collective tragedies do not have an optimal solution.

Another illustration of a collective tragedy in my sense is provided by a group of 32 agents where each agent is in charge of choosing the shape of one face of a polyhedron. The plan of each agent is that the face assigned to her be a hexagon. It is impossible to realize all of the individual plans, on account of Euler polyhedron formula. However, each individual plan is perfectly realizable (actually, 20 out of 32 plans can be realized). I cite this example because it is similar to one used by Visser (1989, 624) to illustrate a point that is similar to the one I want to make:

perhaps there is no true solution [to the semantic paradoxes], maybe we should be content with a number of ways to block the paradox, the choice among which is to be governed by local considerations of utility and simplicity. Maybe language is like the skeleton of Aulonia Hexagona: there is no way to make it what it should be, a regular polygon, without giving up the original plan. (Visser 1989, 624)

I agree that we should take seriously the possibility that there is no true solution to the paradoxes, because no assumption involved in the paradox is more sacred than any of the others. I would add

that there is not even an obligation to choose one among the many, equally legitimate, ways to block the paradox. We can simply adopt the natural reaction and reject the conclusion of the paradox, without indicating at which step the paradoxical reasoning goes wrong.

I want to suggest an analogy between collective tragedies and situations in which a single agent has several plans of actions that are individually consistent but jointly inconsistent. The key is “to think of an agent as a collective” (Rayo 2019, 69) composed of several sub-agents, each with her own plan.

Extending the notion of collective tragedy to cases where a single agent is involved allows us to see the logical/semantic paradoxes as a kind of collective tragedy. The relevant plans of action, in my case, are to respect certain logical/semantic principles: one plan is to reason according to the rules of classical logic, one is to use the truth predicate following the naïve truth rules, one is to accept standard structural rules like cut and contraction, etc. What the paradoxes show is that there is no way to execute all of these plans, i.e. to accept each and every instance of the standard logical and semantic principles without accepting completely implausible conclusions.

However, it is perfectly natural to react to this situation by giving up all of the original plans and replacing each of them with a more modest goal: to apply a logical/semantic principle in most cases, use it as a default rule and be content with reasoning according to a generically valid principle. This would be analogous to the case where a group of agents decides to cover with hexagons only some faces of a polyhedron.

Summing up: I have argued that logical/semantic paradoxes are a kind of collective tragedy. If this is correct, the right reaction to the logical/semantic paradoxes should be the proper reaction in front of a collective tragedy. The right reaction in front of a collective tragedy is to realize that no individual plan is responsible for the tragedy. Analogously, the right reaction in front of the paradoxes is to realize that none of the principles used in the paradoxical reasoning is responsible for the paradox.

#### **4. Wittgenstein and the attitude towards the paradoxes**

Wittgenstein declared that his main goal, in discussing the semantic paradoxes, was to change (what he took to be) the prevailing attitude towards them (see Wittgenstein 1978, III, 82, p. 213). The



attitude that Wittgenstein wanted to contrast was that of seeing the paradoxes as a problem that needs to be fixed, “the superstitious dread...in front of the contradiction”(Wittgenstein 1978, App. III, 17, p. 122); the fear that if our principles have paradoxical consequences, then this makes them useless (see Wittgenstein 1978, VII, 15-16 pp. 374-8). The same attitude drives the attempt to find a proof of consistency, i.e. a guarantee that new paradoxes are not going to arise in the future (see Marconi 1984, section II).

Wittgenstein recommended a different way to look at the paradoxes, according to which the source of the paradoxes is simply our inability to foresee all the possible cases in which the rules governing the use of certain concepts/words might be applied (see for instance Wittgenstein 1978, III 79, p. 207). Marconi (1984, p. 348) summarizes Wittgenstein’s position like this:

The contradiction shows that our concepts are not as well-determined as we would like them to be, i.e., with respect to all their conceivable uses. In certain applications they may turn out to be overdetermined, in others, underdetermined. (Marconi 1984, p. 348)

The relaxed attitude towards the paradoxes that Wittgenstein recommended is nicely captured by the following passage from Hansel Curry (quoted in Marconi 1984, p. 333):

The presentation of the paradoxes...which was made in the universal language, has led many persons to assert that the universal language is inconsistent. So it is, if carelessly used, and carelessness would be expected to cause trouble in any kind of activity. (Curry 1963)

The attitude recommended by Curry resembles very much the attitude of those who take the natural reaction (in the sense of Hofweber 2008, see above) as the right reaction to the paradoxes. Wittgenstein seems to endorse such an attitude when he describes a situation where people discover the paradoxes but don’t get “excited about it” (Wittgenstein 1978 VII 15, p. 376).

I think there is also a striking similarity between the way in which Hofweber formulates the puzzle of the efficacy of the natural reaction discussed in section 2 and Wittgenstein’s insistence, in his discussions with Turing reported in the *Lectures On The Foundations of Mathematics* that the presence of inconsistent rules in a calculus never created any practical problem:

[...] one of the most puzzling features of the paradoxes: why they are in fact no problem whatsoever. No damage has ever been done by them outside of a philosophy department. No planes fell out of the

sky because of them, no money was ever lost, no one was confused into believing that Santa exists because of them. But why not?

(Hofweber 2009, p. 9)

*Turing*: Although you do not know that the bridge will fall if there are no contradictions, yet is almost certain that if there are contradictions it will go wrong somewhere.

*Wittgenstein*: But nothing has ever gone wrong that way yet. And why has it not?

(Wittgenstein 1976, lecture XXII, p. 218)

My main aim in this paper has been to show that there are two simple (minded) arguments to defend (something close to) Wittgenstein's position: one based on the problem of accounting for the success of the natural reaction and one based on the simple observation that the impossibility to consistently combine several plans of action does not show that one of them is more problematic than the others.

There might be arguments different from the ones considered here that support Wittgenstein's position concerning the paradoxes. For instance, one might try to invoke Wittgenstein's rule following considerations to argue that our past use of a rule (of inference) does not determine how we should apply the same rule in the future. However, the rule following considerations are a Pandora's box that I would rather not open. The interest of the present discussion is that we can support (something close to) Wittgenstein's position using argument that do not use any specifically Wittgensteinian assumption.

## 5. Conclusions

In this paper I discussed a neglected puzzle about semantic paradoxes. Semantic paradoxes present us with a puzzling situation: we face an argument that leads to an unacceptable conclusion, but we cannot point out at which step the reasoning goes wrong. This seems to be a big problem: apparently, it shows that some basic principle of reasoning we ordinarily rely upon is invalid and should therefore be given up, but it is not clear which of the principles involved in the paradox is the invalid one.

However, semantic paradoxes present us with a meta-puzzle when we realize that our inability to solve the first puzzle was actually not a big problem. In ordinary reasoning, we seem to have completely ignored the moral of the semantic paradoxes: we did not renounce to any of the principles involved in the Curry or Liar paradox. This *should* have been a problem, but it was not. I argued that

we should give Wittgenstein credit for formulating this meta-paradox, because reflecting on such a meta-paradox is a fruitful exercise.

Reflecting on this meta-paradox might lead us to realize that the semantic paradoxes only show that our logical and semantic principles can not all be strictly valid, but that is not a problem as long as they are generically valid, as argued by Hofweber (2008, 2009). I think this should put some pressure on the idea that the paradoxes show that our ordinary semantic and logical principles are in need of revision, a cherished theme in the literature on conceptual engineering (Scharp 2013). In this paper I have been arguing that our logical and semantic principles do not need to be fixed or replaced with different ones. We can use them in a fruitful way, despite their allegedly problematic features.

The view defended here, that we do not need to replace our logical and semantic principles with new ones, is compatible with the view that exploring ways to replace our logical/semantic principles with new ones is extremely interesting. Shapiro (2014, p. 318) interprets the attempt to replace our ordinary inconsistent logical principles with new consistent ones as an *optimization project*. I have argued that optimization projects are not always mandatory, but I think they are extremely interesting and potentially illuminating. For instance, I am attracted to the view, attributed to Gödel, that the so-called set-theoretical paradoxes show that our ordinary notion of *property* is inconsistent, but they do not show that the mathematical notion of *set* is inconsistent (see Field, 2008, Introduction, for discussion)<sup>1</sup>. It is beyond the scope of this paper to defend this position on the set-theoretical paradoxes, but I think it is worth mentioning it to show that the considerations developed in this paper are not in principle opposed to conceptual engineering.

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<sup>1</sup> Field makes the point in this way: “Kurt Gödel is reported to have remarked ‘There never were any set-theoretic paradoxes, but the property-theoretic paradoxes are still unresolved.’ The idea behind the first part of his remark is presumably that the notion of set was hierarchical from the start, so that it should have been obvious all along that there was no Russell set (in the mathematical sense of ‘set’). The idea behind the second part is that this obvious resolution of Russell’s “paradox” for sets simply doesn’t carry over to the paradox for properties.” (Field 2008, 3)

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